

The quark condensate in relativistic nucleus-nucleus collisions

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Abstract. We compute the modification of the quark condensate $\langle \bar{q}q \rangle$ in relativistic nucleus-nucleus collisions and estimate the 4-volume, where the quark condensate is small ($\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0 \leq 0.1-0.3$) using hadron phase-space distributions obtained with the quark-gluon string model. As a function of the beam energy the 4-volume rises sharply at a beam energy $E_{lab}/A \simeq (2-5)$ GeV, remains roughly constant up to beam energies $\simeq 20$ GeV and rises at higher energies. At low energies the reduction of the condensate is mainly due to baryons, while at higher energies the rise of the 4-volume is due to the abundant mesons produced. Based on our results we expect that moderate beam energies on the order of 10 GeV per nucleon are favourable for studying the restoration of chiral symmetry in a baryon-rich environment in nucleus-nucleus collisions.

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1 Introduction and basic equations

In vacuum the chiral symmetry of QCD is spontaneously broken and the quark condensate $\langle \bar{q}q \rangle$, which is an order parameter of the chiral phase transition, is non-zero. In hadronic matter, the quark condensate is reduced, implying a partial restoration of chiral symmetry, while in quark-gluon matter, beyond the deconfinement transition, one expects chiral symmetry to be restored and consequently the quark condensate to vanish. The leading density and temperature dependence of the condensate has been studied in static (equilibrium) systems [1, 2]. In this letter we estimate the modification of the quark condensate in an inherently dynamic system, namely in a relativistic nucleus-nucleus collision.

The value of the quark condensate in vacuum will be probed by an accurate measurement of the low-energy $\pi\pi$ scattering lengths [3]. However, the in-medium modification of the condensate is, at least presently, not accessible in experiment. Nevertheless, in the present understanding of hot and dense hadronic matter, the restoration of chiral symmetry plays a central role. Consequently, the quark condensate in matter is an essential quantity in the description of such systems. It has been argued that the in-medium masses of hadrons are reduced when the chiral symmetry is partially restored, i.e., when the quark condensate is reduced in magnitude [4]. This idea is supported by QCD sum-rule calculations of vector meson masses in matter [5]. If this idea is correct, the obser-

vation of changes of hadron masses may open an indirect way of exploring the restoration of chiral symmetry in dense matter.

The low-density and low-temperature behaviour of $\langle \bar{q}q \rangle$ is governed by the relation [1, 2],

$$\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0} = 1 - \sum_h \frac{\sigma_h \rho_h^s}{f_\pi^2 m_\pi^2}, \quad (1)$$

where the sum runs over hadron species h . Here σ_h denotes the σ -commutator of the relevant hadron, ρ_h^s the corresponding scalar density of non-interacting particles, $f_\pi = 94$ MeV the pion decay constant and m_π the pion mass.

For nuclear matter at zero temperature and low baryon density ρ , the leading term in Eq. (1) is given by a gas of non-interacting nucleons. Since, in the rest frame of the system, the nucleon scalar density approximately equals the baryon density, one finds [2] using $\sigma_N = 45$ MeV (see ref. [6])

$$\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0} \simeq 1 - \frac{\rho}{3\rho_0}, \quad (2)$$

which, if higher-order terms are neglected, implies that the quark condensate is reduced by about one third at normal nuclear matter density $\rho_0 = 0.16 \text{ fm}^{-3}$. The terms of higher order in the density have been estimated in relativistic Brueckner-Hartree-Fock calculations using realistic nucleon-nucleon interactions [7–9]. Qualitatively

these calculations show that the quark condensate is indeed reduced approximately linearly with density up to $\rho \sim 1.5\rho_0$. At somewhat higher densities the condensate seems to stay at a value of about 40 % of its vacuum value [9]. We can account for this effect in an approximate manner by requiring that the reduction of $\langle\bar{q}q\rangle/\langle\bar{q}q\rangle_0$ due to baryons should not exceed 60%. Eventually, at least at densities where quark degrees of freedom become relevant, one expects chiral symmetry to be restored and consequently the condensate to vanish. However, at beam energies, where such densities could be reached, the meson densities are large and dominate. Consequently, the precise behaviour of the modification of the quark condensate due to baryons at very large baryon densities is not expected to change our results qualitatively.

At low temperatures and zero net baryon density, the modification of the quark condensate in Eq. (1) is dominated by pions. In the chiral limit, where the explicit symmetry breaking vanishes and $m_\pi \rightarrow 0$, the relative change of the quark condensate in a pion gas is given by [1]

$$\frac{\langle\bar{q}q\rangle}{\langle\bar{q}q\rangle_0} = 1 - \frac{T^2}{8f_\pi^2} \quad (3)$$

to leading order. Since two- and three-loop contributions do not significantly change the temperature dependence of the quark condensate [1, 10], we feel that it is justified to neglect higher-order terms in (1) for pions and other mesons.

Consequently, we employ relation (1) with a finite pion mass and a slightly modified form thereof, which accounts for the higher-order terms in the baryon density as discussed above. Further details are given in Sect. 3, where the implementation in the transport calculation is discussed.

While it may be possible to describe the final stages of nucleus-nucleus collisions approximately with equilibrium thermodynamics this is clearly not possible for the early stages. The non-equilibrium character of nucleus-nucleus collisions can be taken into account by evaluating the scalar densities in Eq. (1) with the corresponding, possibly non-equilibrium, phase-space densities, obtained using a model for the space-time evolution of nucleus-nucleus collisions. Before we turn to this problem, we present some qualitative arguments for the behaviour of the quark condensate in heavy-ion collisions.

2 Qualitative consideration of heavy-ion collisions

In contrast to equilibrium systems, where the quark condensate has been studied so far, nucleus-nucleus collisions are characterized by a short time scale on the order of 10 fm/c. So far the typical time scale τ for the quark condensate to respond to changes of the medium has not been studied. However, one would expect this time scale to be governed by the mass of the lightest scalar meson and thus to be of the same order as the hadronic time scales, i.e. $\tau \lesssim 1$ fm/c. In this exploratory calculation we

assume that the response of the condensate is instantaneous, except for produced particles, whose contribution to the quark condensate is taken into account only after a formation (proper) time $\tau_f = 1$ fm/c.

Before we turn to nucleus-nucleus collisions, let us consider the quark condensate in a moving nucleus. Since the quark condensate is a Lorentz scalar, it must be independent of the reference frame of the observer. To illustrate this, we repeat the lowest-order calculation [2], which leads to Eq. (1). The leading contribution to the energy density of a system of nucleons is given by

$$\varepsilon = \int d^3p \sqrt{\mathbf{p}^2 + m_N^2} n_{\mathbf{p}}, \quad (4)$$

where $n_{\mathbf{p}}$ is the nucleon distribution function. The modification of the quark condensate due to the presence of the nucleons is, according to the Feynman-Hellmann theorem,

$$\langle\bar{q}q\rangle - \langle\bar{q}q\rangle_0 = \frac{d\varepsilon}{dm_q} = \frac{\sigma_N}{m_q} \int d^3p \frac{m_N}{\sqrt{\mathbf{p}^2 + m_N^2}} n_{\mathbf{p}}, \quad (5)$$

where $\sigma_N = m_q dm_N/dm_q$. The integral on the right-hand side of (5) is the scalar nucleon density. Since $d^3p/\sqrt{\mathbf{p}^2 + m_N^2}$ and $n_{\mathbf{p}}$ are Lorentz invariants, the value of the scalar density and consequently the quark condensate are independent of the reference frame¹.

It follows that the condensate in the interior of a moving nucleus equals the condensate in a nucleus at rest, i.e., if the lowest-order approximation (2) is valid, it is reduced by about 1/3 from its vacuum value. This means that when two nuclei pass through each other without interacting, the quark condensate in the overlap region is reduced by the amount corresponding to nuclear matter at $\rho = 2\rho_0$, i.e., by about 2/3. On the other hand, the baryon density in a moving system is enhanced by Lorentz contraction of the volume $\rho = 2\gamma\rho_0$, where $\gamma = 1/\sqrt{1-v^2}$ and v is the velocity of the nucleus. At AGS energies this purely kinematical effect without any dynamical compression yields a baryon density of (5-6) ρ_0 , while at CERN energies one finds about 20 ρ_0 . Similarly, also the energy density is strongly enhanced partly due to the Lorentz contraction of the collision volume. This trivial Lorentz contraction, which is responsible for a substantial fraction of the very large baryon and energy densities found in simulations of heavy-ion collisions at ultra-relativistic energies, does not affect the quark condensate. Consequently, the baryon density is not a suitable measure for the restoration of chiral symmetry in such collisions.

When interactions are taken into account, the nucleons are slowed down and other hadrons (mesons and baryon-antibaryon pairs) are produced. Both effects reduce the quark condensate. In order to illustrate this we consider the Landau model for hadronic collisions [11]. If, as assumed in this model, the deceleration is very fast, the interaction volume is not expected to differ appreciably

¹ We note that these arguments are not restricted to equilibrium systems; the distribution function $n_{\mathbf{p}}$ in (4) and (5) is arbitrary.

from the Lorentz contracted collision volume. Thus, after stopping there is a large density ($\approx 2\gamma\rho_0$) of relatively slow hadrons in this volume, which can lead to a considerable reduction of the quark condensate. For the stopped nucleons we have $m_N/\sqrt{p^2+m_N^2} \sim 1$, so that their contribution to the scalar density approximately equals the corresponding part of the baryon density (cf. Eq. (5)). Similarly, for the bulk of the produced hadrons, the velocities are relatively small, so that the associated scalar densities are well approximated by the particle densities. This simple consideration shows, that the value of the quark condensate in nucleus-nucleus collisions depends sensitively on the collision dynamics, in particular on stopping and particle production rates.

We note that when the finite sizes of the hadrons are taken seriously, one arrives at similar conclusions. The relevant quantity, which determines the quark condensate of such a system is not the number density but the volume fraction which is not occupied by hadrons [12]. In the spirit of the chiral bag model it is assumed that the interior of a nucleon is in the chirally symmetric phase, and consequently that the quark condensate there is effectively zero. Thus, each nucleon represents a small volume, where the chiral symmetry is locally restored, and the average value of the condensate in nuclear matter is proportional to the fraction of the volume which is unoccupied. By identifying the coefficient of the term linear in density with that given by Eq. (1), one finds for the volume of a nucleon at rest $v_N = \sigma_N/f_\pi^2 m_\pi^2$, which implies $R_N = 0.8$ fm. Now consider a moving nucleus composed of finite-size nucleons. Since all volumes, that of the nucleus and those of the nucleons, are Lorentz contracted by the same factor², the fraction of the volume which is not occupied is a Lorentz invariant. On the other hand, a nucleon, which is stopped in a nucleus-nucleus collision, recovers its rest volume. Consequently, stopping leads to an increase in the occupied fraction of the Lorentz contracted interaction volume and thus to a decrease of $|\langle\bar{q}q\rangle|$. The occupied fraction is increased further by the produced hadrons. Thus, this simple picture provides an intuitive interpretation of the features discussed above.

3 The quark condensate in Au on Au collisions

From the arguments presented in Sect. 2 it should be clear that a realistic estimate of the quark condensate in relativistic nucleus-nucleus collisions can be obtained only by using a reliable model for the space-time evolution of the collision. We employ the Quark-Gluon String Model (QGSM) [13], which is based on the string phenomenology of hadronic interactions. Baryons and mesons belonging to the two lowest $SU(3)$ multiplets along with their antiparticles are included. The interactions between the hadrons are described by a collision term, where the Pauli principle

² In this argument we assume that the internal velocities v_i of the nucleons in the nucleus are small compared to the velocity v of the nucleus, so that terms of order $(v_i/v)^2$ are negligible.

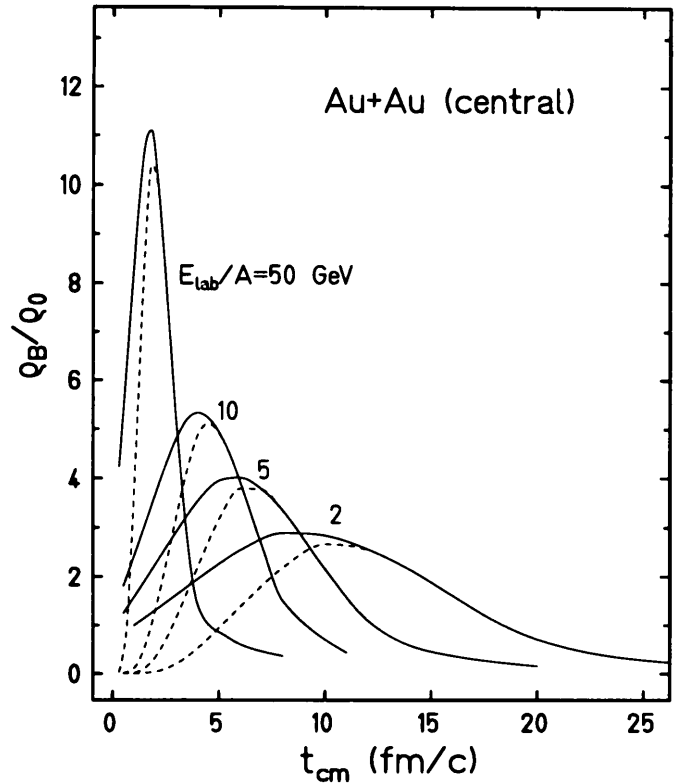


Fig. 1. Time evolution of the central baryon density in head-on Au+Au collisions for beam energies $2 \text{ GeV} \leq E_{lab}/A \leq 50 \text{ GeV}$. The solid lines correspond to all baryons while the dashed lines are for the participants only

is imposed in the final states. This includes elastic collisions as well as hadron production and decay processes. As mentioned above, a proper time $\tau_f = 1$ fm/c for the formation of hadrons is incorporated. Mean fields and interactions between strings are not taken into account in the present version of the model. The general experimental characteristics of relativistic nucleus-nucleus collisions are well reproduced over a large range of beam energies from SIS [14], to AGS [15] and SPS energies [13,16]. Thus, we expect that the model describes the space-time evolution of such collisions reasonably well.

In Fig. 1 we show the time evolution of the central baryon density for head-on Au+Au collisions at beam energies from $E_{lab}/A = 2$ GeV up to 50 GeV. The initial time ($t = 0$) corresponds to the instant when the two Lorentz-contracted nuclei touch. The densities are obtained from the number of particles in the cylindrical test volume of radius R_f and length $2R_f/\gamma_{cm}$ at rest in the center-of-mass frame. The factor $1/\gamma_{cm}$ in the longitudinal direction accounts for the Lorentz contraction of the colliding nuclei. The total baryon density in the test volume increases rapidly reaching a maximum shortly after the point of maximum overlap. Subsequently the density decreases rapidly with time. We also show the density of participant nucleons, defined as those that have suffered at least one collision. The maximum baryon density reached at $E_{lab}/A = 10$ GeV is in good agreement with that found

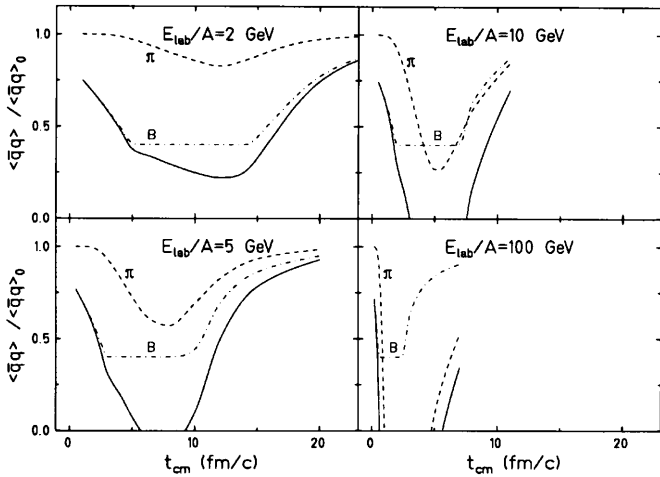


Fig. 2. Time evolution of the central quark condensate (*solid lines*) in head-on Au+Au collisions for $E_{lab}/A = 2, 5, 10, 100$ GeV. The contribution from baryons (*dash-dotted lines*) and pions (*dotted lines*) are shown separately

in ref. [17]. The maximum density is reached at the same time, $t \approx 4$ fm/c, while our maximum value is somewhat smaller due to a larger test volume; we use $R_f = 5$ fm while in [17] a sphere of radius 2 fm was employed.

A comparison of the total and participant densities shows that even near the maximum not all nucleons have experienced a collision. Nevertheless, both definitions of the density imply that very large baryon densities are reached at high beam energies. However, as we argued above, these densities are not immediately relevant for the restoration of chiral symmetry and associated medium effects.

We compute the quark condensate in a given volume V by implementing Eq. (1) in the transport model in the form

$$\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0} = 1 - \frac{1}{V} \sum_{i=1}^{N_V} \frac{m_i}{\epsilon_i} \frac{\sigma_i}{f_\pi^2 m_\pi^2}, \quad (6)$$

where the sum runs over all particles N_V in V . Here σ_i , m_i and $\epsilon_i = \sqrt{\mathbf{p}_i^2 + m_i^2}$ denote the sigma term, mass and energy of particle i . For nucleons and pions we use $\sigma_N = 45$ MeV and $\sigma_\pi = m_\pi/2$, respectively, while for all other hadrons we take the sigma term to be given by $\sigma_i = (Q_i/Q_N)\sigma_N$, where Q_i denotes the light-valence-quark content of hadron i . Produced particles are counted only after their formation time $(\epsilon_i/m_i)\tau_f$ has passed. As mentioned above we approximate higher-order effects by limiting the modification of the quark condensate due to baryons to be less than 0.6, cf. [8,9,12].

The resulting time evolution of the quark condensate is shown in Fig. 2 for the same test volume as for the baryon densities in Fig. 1. For beam energies beyond 4 GeV, the ratio $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0$ becomes negative for some time during the collision. Clearly, the low-density approximation is no longer valid when this happens. However, if we are interested only in the time Δt_{qc} that the system spends in a state where the quark condensate is small,

say $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0 \leq 0.3$, this approximation is expected to be reasonable. Indeed, in this case only the reliability of the approximation for $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0 > 0.3$ matters. We note at this point that possible modifications of the dynamics, e.g. due to an equation of state with a strong first-order phase transition, are not included in the transport model. However, such effects are not expected to change the expansion time scale dramatically [18].

At a beam energy of $E_{lab}/A = 2$ GeV the pion density remains relatively small, and hence the main reduction of $|\langle \bar{q}q \rangle|$ is due to baryons which in our model reach densities of $3\rho_0$ (see Fig. 1). At higher energies, the pion contribution becomes gradually more important. At $E_{lab}/A = 50$ GeV and above the reduction of $|\langle \bar{q}q \rangle|$ is dominated by pions and other mesons. We note that the time period Δt_{qc} , where $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0 < r$, with $r = 0.1 - 0.3$, in the cylindrical test volume is between 5 and 7 fm/c for all bombarding energies beyond 5 GeV. At SIS energies and partially also at AGS energies, the reduction of the quark condensate is induced predominantly by the baryons, while at ultra-relativistic energies the meson contribution prevails.

In general we expect that observable effects due to partial restoration of chiral symmetry should depend not only on the time, where the quark condensate is small, but also on the corresponding volume. Clearly, in order to have a clean signal one would like the time to be as long as possible, and in order to minimize unwanted surface effects the volume should be as large as possible. However, the relative importance of time and volume probably depends on the particular signal one considers and is not known in general. Consequently, an optimal criterion of universal validity is difficult if not impossible to construct. We choose a relatively simple quantity that accounts for both time and volume, the 4-volume

$$\Omega_r = \int d^3x dt \theta \left(r - \frac{\langle \bar{q}q \rangle(x,t)}{\langle \bar{q}q \rangle_0} \right) \quad (7)$$

$$\simeq \int dt \left[\sum_i \Delta V_i \theta \left(r - \frac{\langle \bar{q}q \rangle_i(t)}{\langle \bar{q}q \rangle_0} \right) \right], \quad (8)$$

where θ denotes the Heaviside function. In the second line we indicate how Ω_r is computed in the transport model. Using Eq. (6) we evaluate the quark condensate $\langle \bar{q}q \rangle_i(t)$ in each cell i at a given time t , and then sum the volumes ΔV_i of those cells where $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0$ is smaller than r . Finally we integrate over time. Thus the 4-volume, where the quark condensate is small, is determined dynamically. We make use of the cylindrical symmetry of central collisions to reduce the number of cells. The cells are rings around the collision axis of longitudinal and radial extension $\Delta z = 2$ fm and $\Delta r = 1$ fm, respectively.

The 4-volume Ω_r is shown by the full lines in Fig. 3 as a function of the bombarding energy for different values of r . Here we employ the modified version of Eq. (1), where the baryon contribution to the quark condensate is limited. We find a sharp increase at a threshold energy of a few GeV per nucleon, a plateau in an intermediate energy range up to ~ 20 GeV per nucleon and an increasing 4-volume for energies beyond that. The threshold and

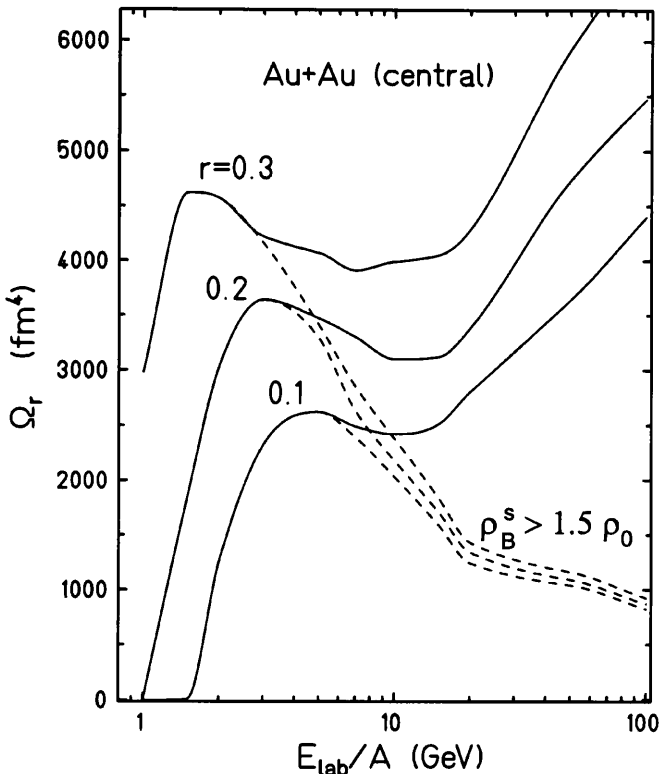


Fig. 3. The invariant 4-volume Ω_r (solid line), where $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0$ drops below $r = 0.1, 0.2$ and 0.3 , as function of the beam energy. The corresponding 4-volume, where in addition the scalar baryon density exceeds $1.5 \rho_0$, is shown by the dashed lines

the plateau regions are associated with high baryon density, while the rise at high energies is due largely to meson degrees of freedom. At this point we note that the saturation of the baryon contribution, introduced to account for higher-order effects, plays a crucial role at low energies but does not affect the 4-volume appreciably at high beam energies, $E_{lab} \geq 3$ GeV.

In order to obtain a quantitative characterization of the different regimes we also show the 4-volume, where the quark condensate is small with the additional constraint that the baryon scalar density should be larger than $1.5\rho_0$ (dashed lines in Fig. 3). This 4-volume exhibits a maximum at a relatively low beam energy, and then decreases as the energy is increased. The reduction of the contribution from baryon-rich matter is basically due to the shortening of the corresponding time scale (Fig. 2). This in turn is a result of the higher expansion velocity, which leads to a faster dilution of the baryon density (cf. Fig. 1). Since the net baryon number is constrained by conservation laws, a more rapid expansion cannot be compensated for by an enhanced rate for particle production. On the other hand, the growth of the meson contribution to the 4-volume at high energies is due to a high rate for meson production, which compensates for the strong expansion and implies that a dense meson gas exists over a relatively long time scale in an expanding interaction volume.

The characteristics of the restoration of chiral symmetry at high baryon density may differ from that at high temperature. At zero baryon density and finite temperatures, the pion is the most abundant hadron. The thermal pions induce a modification of the quark condensate to lowest order, which in the chiral limit gives rise to the well known T^2 term. However, because the pion is a Goldstone boson, and consequently its interaction terms involve gradients, the leading correction to e.g. the mass of the ρ meson in a pion gas [21] is $\mathcal{O}(T^4)$. Thus, the modification of meson masses as a function of temperature is not simply related to the quark condensate. On the other hand, at zero temperature and finite baryon density, the leading corrections to the quark condensate and hadronic masses are due to nucleons. Here the situation may be different, since e.g. non-gradient ρ -nucleon couplings are allowed³.

At this point we also note that the in-medium modifications invoked in interpretations of the low-mass lepton-pairs in ultra-relativistic nucleus-nucleus collisions at the SPS [19,20] depend mainly on high baryon densities [23–25]. For such medium effects the dashed curves in Fig. 3 and the baryon time scales of Fig. 2 are relevant. Consequently, if such a model is correct, one expects an even more pronounced enhancement of lepton pairs at lower beam energies, say $E_{lab}/A \sim 10$ GeV, where high baryon densities dominate.

In the baryon-rich case, the 4-volume exhibits a maximum, which – depending on the value of r – lies in the range 2–8 GeV. In principle this indicates the optimal beam energy for exploring effects connected with the restoration of chiral symmetry associated with high baryon densities in heavy-ion collisions. However, the quark condensate is not a direct observable. Furthermore, as noted above, the relative weight of the time and volume depends on the probe under consideration. Consequently, there is at best an indirect connection between the maximum in the 4-volume and the optimal energy for a given probe.

However, the threshold behaviour of the 4-volume is a characteristic feature, which we expect to be of general validity for any probe of the restoration of chiral symmetry in dense and hot matter. The threshold energy for a given observable may differ from our estimate $E_{lab}/A \simeq (2-4)$ GeV, due to the unknown relative importance of the time and volume and additional uncertainties, like e.g. the response time of a given probe to changes in the quark condensate. Nevertheless, our results indicate that one should expect significant effects due to partial restoration of chiral symmetry in a large volume over an extended period of time already at moderate beam energies, say $2 \text{ GeV} < E_{lab}/A < 10 \text{ GeV}$.

4 Conclusions

We argue that the quark condensate is better suited than the baryon density as a measure for chiral restoration in simulations of relativistic nucleus-nucleus collisions. The

³ See however ref. [22] for a critical discussion of this point.

condensate is directly related to the spontaneous breaking of chiral symmetry, since it is an order parameter for this transition. Furthermore, it is not encumbered by trivial Lorentz contraction effects and accounts for both baryons and mesons in a natural way.

We have estimated the quark condensate in relativistic nucleus-nucleus collisions. We find that the invariant 4-volume, where the condensate is small, increases rapidly at $E_{lab}/A = (2 - 5)$ GeV, levels off at intermediate energies and increases for beam energies beyond ~ 20 GeV. This behaviour is due to an interplay between meson and baryon contributions to the quark condensate. For baryon-rich matter, the 4-volume decreases at high energies with increasing beam energy, giving rise to a maximum at fairly low beam energies. Although for a given probe the optimal beam energy may vary, experimental signatures of chiral symmetry restoration are expected to exhibit a characteristic threshold behavior corresponding to the sharp rise of the 4-volume. All in all, our results indicate that the conditions reached in nucleus-nucleus collisions at moderate beam energies, $2 \text{ GeV} < E_{lab}/A < 10 \text{ GeV}$, are favourable for exploring the restoration of chiral symmetry in dense/hot matter. For probes that rely on high baryon densities, the optimal conditions are probably reached at beam energies around $E_{lab}/A \sim 10 \text{ GeV}$.

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